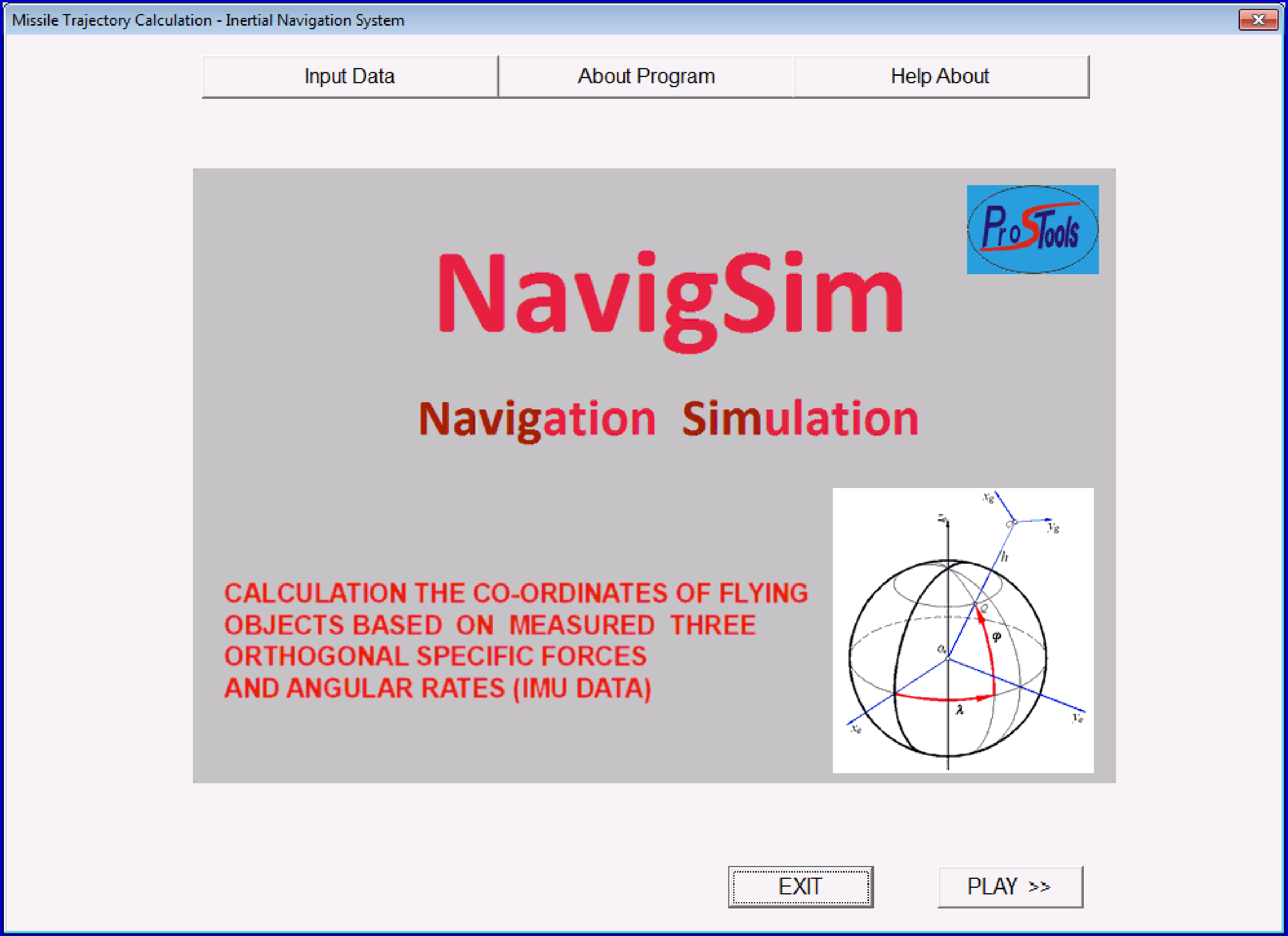


Missile Trajectory Calculation - Inertial Navigation System

NavigSim

Navigation Simulation

CALCULATION THE CO-ORDINATES OF FLYING
OBJECTS BASED ON MEASURED THREE
ORTHOGONAL SPECIFIC FORCES
AND ANGULAR RATES (IMU DATA)

version 2.0 Sep. 2010

SHORT DESCRIPTION

- PROGRAM NUMERICALLY SOLVES (INTEGRATES) NAVIGATION AND LATITUDE EQUATIONS FOR A GIVEN INITIAL CONDITIONS AND MEASURED (OR NUMERICALLY SIMULATED) DATA WHICH CONSISTS OF THREE CO-ORDINATES OF ORTHOGONAL SPECIFIC FORCE OBTAINED FROM ACCELEROMETERS AND THREE CO-ORDINATES OF ANGULAR RATES OBTAINED FROM RATE GYROS.
- PROGRAM CAN CALCULATE ESTIMATES OF THE STANDARD DEVIATIONS OF VEHICLE CO-ORDINATES DUE TO SENSORS BIAS ERRORS AND DUE TO SENSORS IN-RUN ERRORS (PROPAGATION OF ERRORS IN TIME) BY THE MONTE-CARLO METHOD. MISSILE LATITUDE IS DEFINED BY QUATERNIONS. NAVIGATION EQUATIONS ARE EXPRESSED IN LOCAL GEOGRAPHIC CO-ORDINATE AXIS SYSTEM.

PURPOSE

- THE PURPOSE OF THE PROGRAM IS CALCULATION OF CO-ORDINATES OF FLYING VEHICLE BASED ON MEASURED THREE ORTHOGONAL SPECIFIC FORCES AND ANGULAR RATES.
- PROGRAM CALCULATES TRAJECTORY OF A MISSILE BASED ON STRAPDOWN INERTIAL NAVIGATION THEORY. IT CAN BE USED TO DETERMINE COORDINATES OF THE FLYING OBJECTS OR TO TEST VARIOUS NAVIGATION ALGORITHM, VARIOUS TYPES OF VEHICLE MOTION, COMPUTATION METHODS AND INFLUENCE OF VARIOUS TYPES OF INSTRUMENT NOISES ON THE DISPERSION OF TRAJECTORIES

PROGRAM SERVES FOR:

- SOLVING NAVIGATION PROBLEM,
- ESTIMATION OF MISSILE ERROR ON TARGET DUE TO ERRORS OF SENSORS IN INERTIAL MEASUREMENT UNIT,
- TESTING OF VARIOUS ALGORITHMS FOR SOLVING NAVIGATION AND LATITUDE EQUATIONS,
- TESTING THE INFLUENCE OF SAMPLING RATE ON THE ACCURACY OF INTEGRATION OF EQUATIONS NAVIGATION AND LATITUDE EQUATIONS,
- TESTING THE INFLUENCE OF THE PARAMETERS OF THE EARTH MODEL ON THE ACCURACY OF CALCULATION OF COORDINATES,
- TESTING THE INFLUENCE OF THE VARIOUS TYPES OF MOTION (CONNING MOTION, SCULLING MOTION) ON THE ACCURACY OF THE CALCULATION OF COORDINATES

HELP ABOUT

MISSILE TRAJECTORY CALCULATION
BY INERTIAL NAVIGATION SYSTEM

Version 1.0

Copyright © 2009 Miodrag Ćurčin PhD,
Belgrade, Serbia

All Rights Reserved

INPUT FILES

Input data are located into two files:

1) First file contains input data and parameters for defining a calculation. This file can be chosen through an open dialog box, or can be generated by typing data into proper edit boxes.

2) Second file is an INS data file with measured values. This data are going to be used when a "Measured values" radio button is selected.

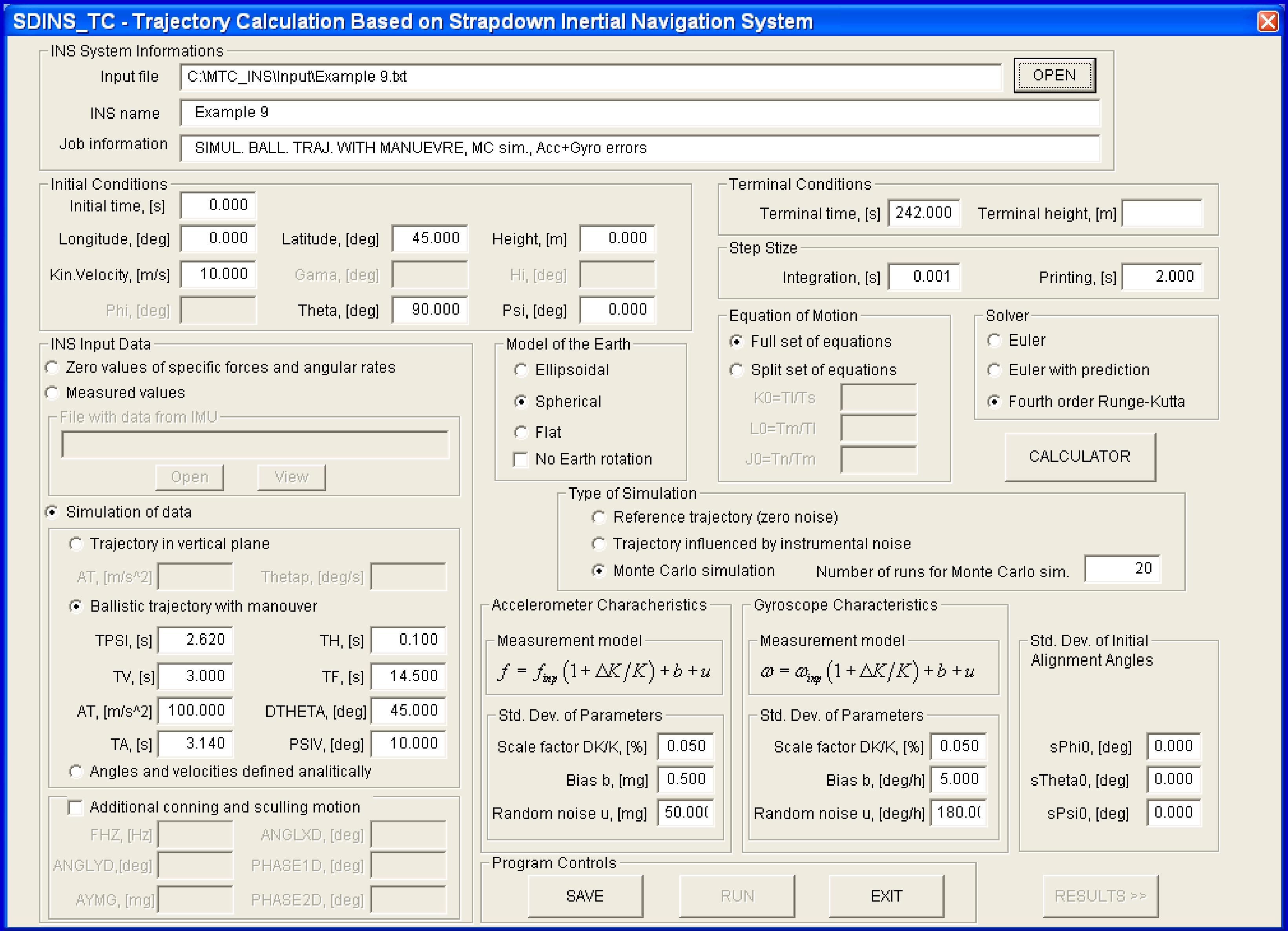
INPUT DATA FILE

```
TEST_V3                          - title
SPLIT METHOD                     - subtitle
0.000    1.000                   - time_ini, time_end
2   3   1                        - INSdatamode, integ_method, modesplit
1      10       5                - K0, L0, J0
1  1                             - mEarth_R, mEarth_rot
-1                               - NMC
0.000    0.000    0.000          - sdG
0.000    0.000    0.000          - sdA
0.000    0.000    0.000          - sdAtt
0.001    0.001                   - hinteg0, hprint0
45.000    0.000    0.000         - alat0s, along0s, height0
3                                - ITRAJTYPE
1                                - HAR
50.000  0.100  0.100   90.000 10000.00  0.000          - FHz,ANGLXD,
                                       ANGLYD,PHASE1D,AYMG,PHASE2D
```

INS DATA FILE WITH MEASURED VALUES

```
50.00
     .000000000    .000000000    .000000000   47.823214206    .118012700    .000000000
     .000000000   -.000000122   -.001232470   47.843629280    .118305927   -.000040395
     .000000000   -.004135241   -.002470336   47.864058733    .119147372   -.287137428
     .000000000   -.008270488   -.003718723   47.884502562    .120434460   -.287135858
     .000000000   -.012405634   -.004981808   47.904960765    .122088169   -.287113638
     .000000000   -.016540427   -.006263033   47.925433338    .124046551   -.287068169
     .000000000   -.020674599   -.007565257   47.945920277    .126260259   -.286996675
     .000000000   -.024807860   -.008890874   47.966421577    .128689405   -.286896196
     .000000000   -.028939897   -.010241896   47.986937235    .131301306   -.286763588
     .000000000   -.033070378   -.011620020   48.007467245    .134068836   -.286595518
```

INPUT DATA MENU



SDINS_TC - Trajectory Calculation Based on Strapdown Inertial Navigation System

INS System Informations
Input file: C:\MTC_INS\Input\Example 9.txt
INS name: Example 9
Job information: SIMUL. BALL. TRAJ. WITH MANUEVRE, MC sim., Acc+Gyro errors

Initial Conditions
Initial time, [s]: 0.000
Longitude, [deg]: 0.000
Latitude, [deg]: 45.000
Height, [m]: 0.000
Kin.Velocity, [m/s]: 10.000
Gama, [deg]:
Hi, [deg]:
Phi, [deg]:
Theta, [deg]: 90.000
Psi, [deg]: 0.000

Terminal Conditions
Terminal time, [s]: 242.000
Terminal height, [m]:

Step Stize
Integration, [s]: 0.001
Printing, [s]: 2.000

Equation of Motion
● Full set of equations
○ Split set of equations
K0=Tl/Ts:
L0=Tm/Tl:
J0=Tn/Tm:

Solver
○ Euler
○ Euler with prediction
● Fourth order Runge-Kutta

INS Input Data
○ Zero values of specific forces and angular rates
○ Measured values
File with data from IMU:
● Simulation of data
○ Trajectory in vertical plane
AT, [m/s^2]:
Thetap, [deg/s]:
● Ballistic trajectory with manouver
TPSI, [s]: 2.620
TH, [s]: 0.100
TV, [s]: 3.000
TF, [s]: 14.500
AT, [m/s^2]: 100.000
DTHETA, [deg]: 45.000
TA, [s]: 3.140
PSIV, [deg]: 10.000
○ Angles and velocities defined analitically
☐ Additional conning and sculling motion
FHZ, [Hz]:
ANGLXD, [deg]:
ANGLYD,[deg]:
PHASE1D, [deg]:
AYMG, [mg]:
PHASE2D, [deg]:

Model of the Earth
○ Ellipsoidal
● Spherical
○ Flat
☐ No Earth rotation

Type of Simulation
○ Reference trajectory (zero noise)
○ Trajectory influenced by instrumental noise
● Monte Carlo simulation
Number of runs for Monte Carlo sim.: 20

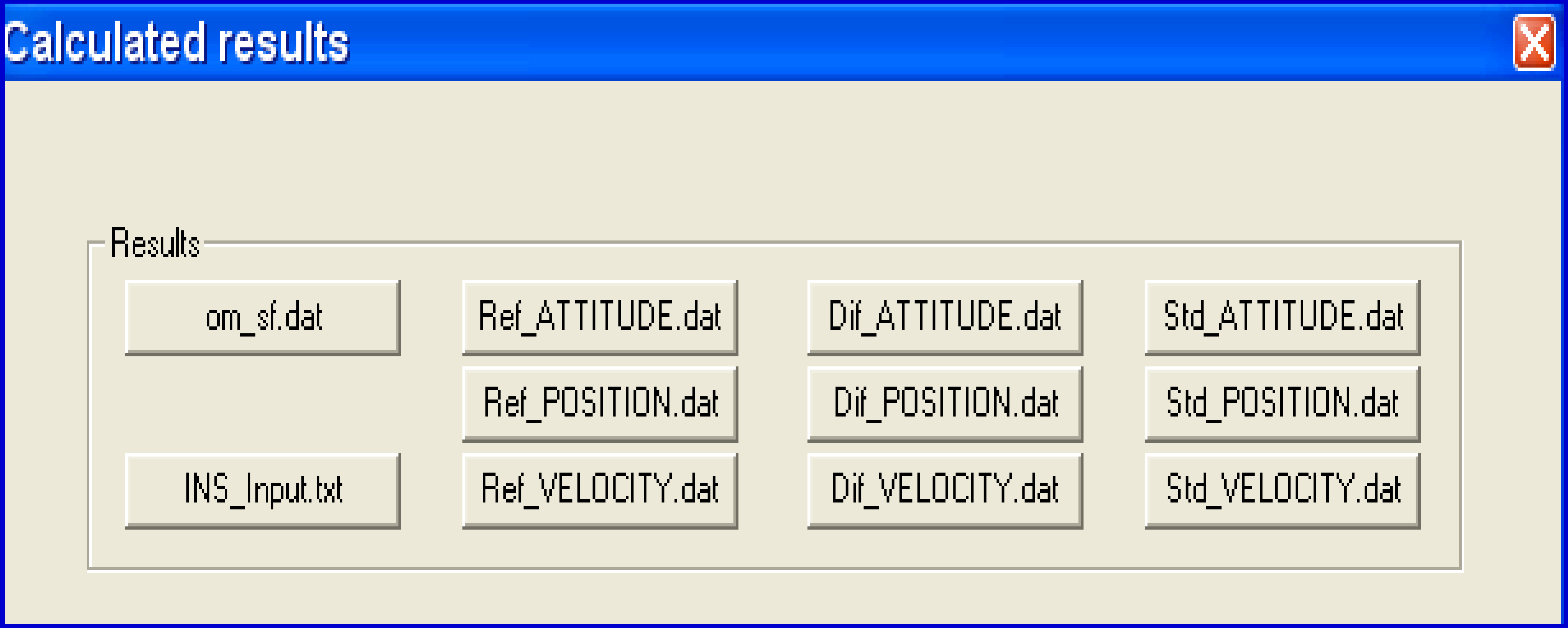
Accelerometer Characheristics
Measurement model: $f = f_{inp}\left(1+\Delta K/K\right)+b+u$
Std. Dev. of Parameters
Scale factor DK/K, [%]: 0.050
Bias b, [mg]: 0.500
Random noise u, [mg]: 50.00(

Gyroscope Characteristics
Measurement model: $\omega = \omega_{inp}\left(1+\Delta K/K\right)+b+u$
Std. Dev. of Parameters
Scale factor DK/K, [%]: 0.050
Bias b, [deg/h]: 5.000
Random noise u, [deg/h]: 180.0(

Std. Dev. of Initial Alignment Angles
sPhi0, [deg]: 0.000
sTheta0, [deg]: 0.000
sPsi0, [deg]: 0.000

Program Controls

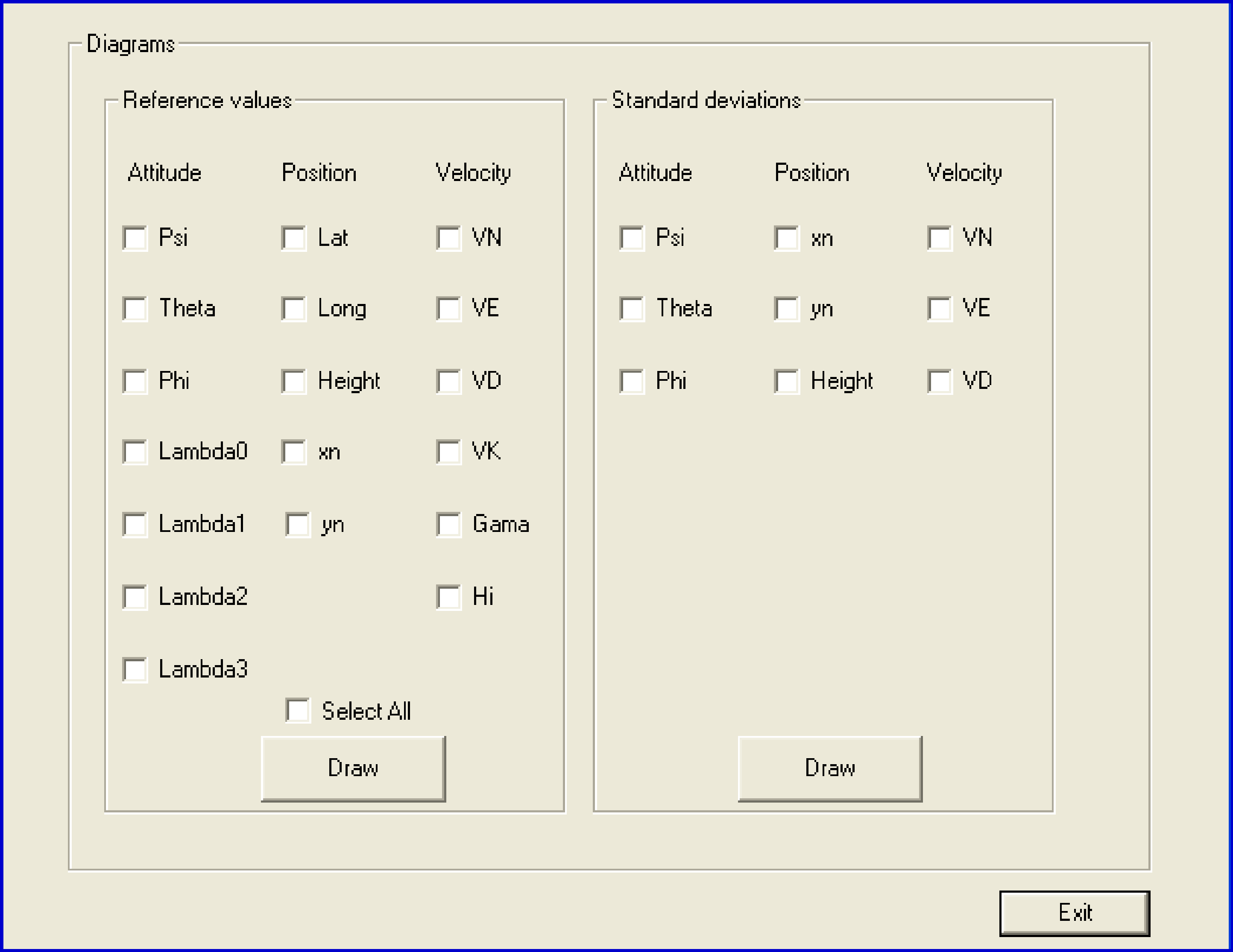
OUTPUT FILES



OUTPUT FILES

- File with calculated or measured angular rates and specific forces
- Files with calculated attitude parameters, coordinates of the missile centre of mass and velocities on reference trajectory with respect to time of flight.
- File with calculated estimates of the statistics of attitude parameters, coordinates of the missile centre of mass and velocities with respect to time of flight.

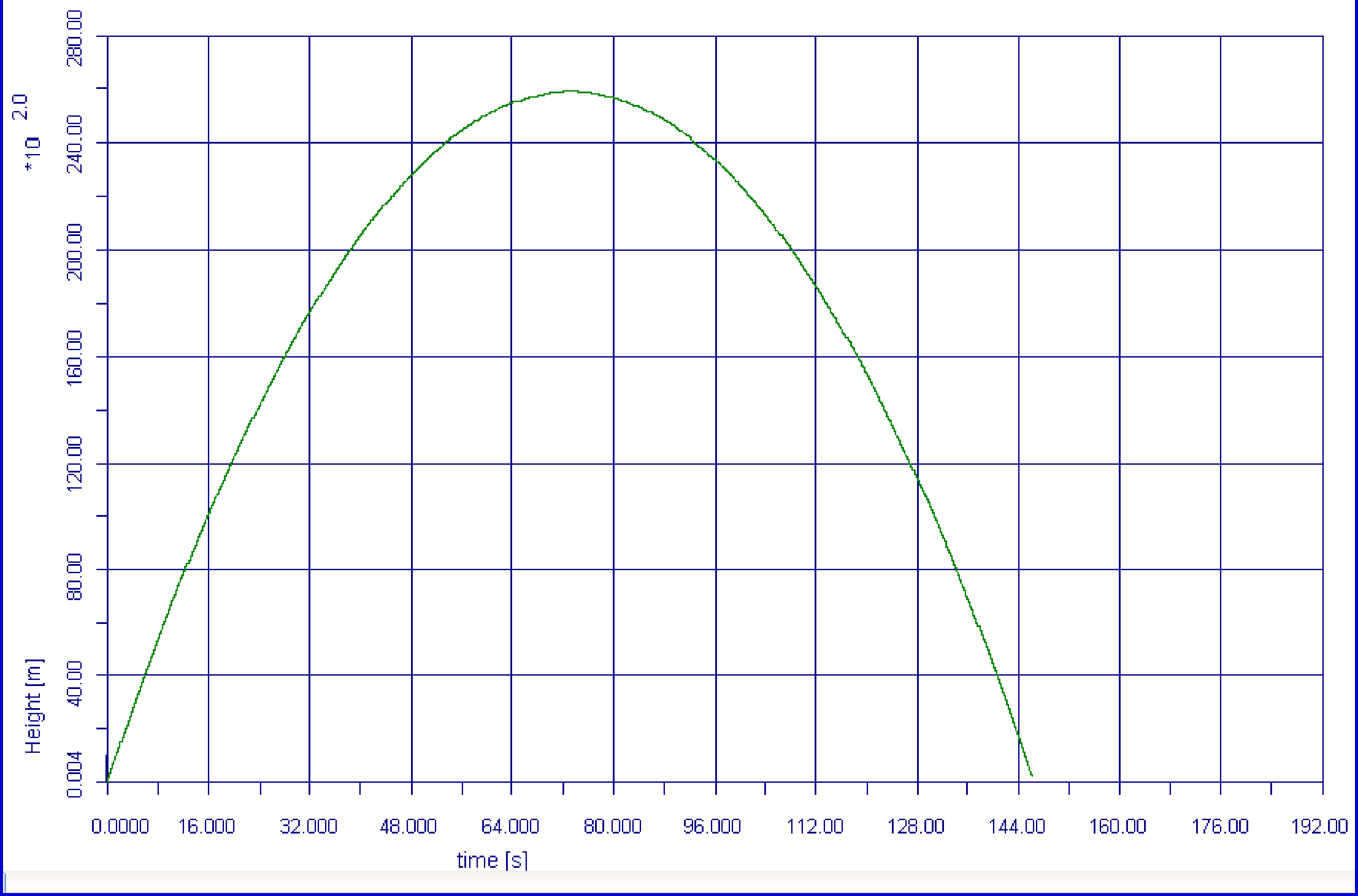
DIALOG FOR CREATING DIAGRAMS



EXAMPLE 1

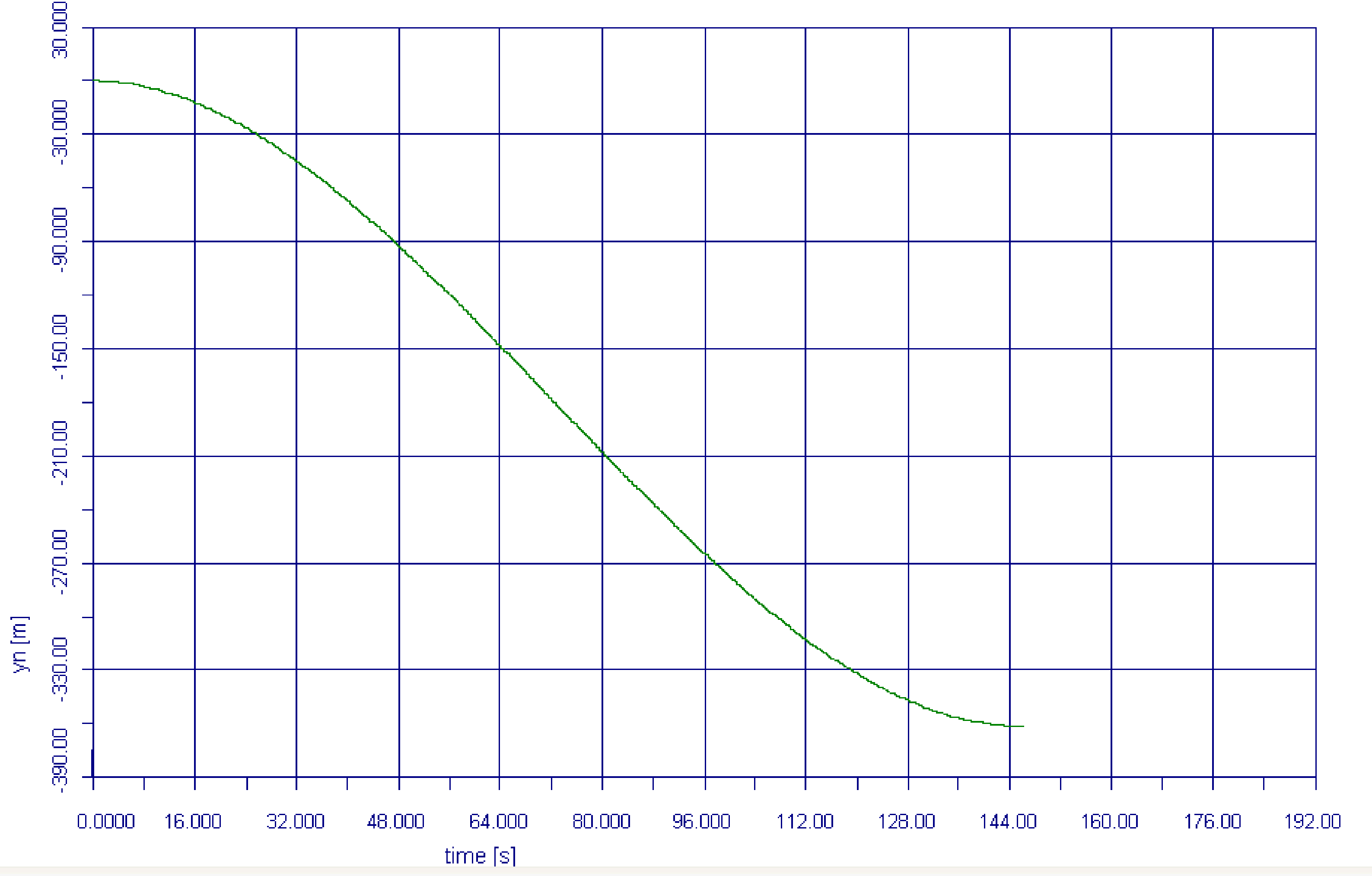
- Firing directly to the North, and examine influence of Earth rotation.
- In this example we use ellipsoidal model of the Earth. A pure ballistic trajectory simulation was done – zero values for angular rates and specific forces. For equation model a full set of equations was used. Initial azimuth was zero, and initial flight-path angle was 45 degree. Initial geographics height was equal to zero, and initial velocity was 1000 m/s.
- Due to Earth rotation geographic co-ordinate of impact in the East direction appears. Trajectory was finished at y_n=-361 m. The minus sign means that the impact was shifted to the West, although we fired directly to the Nord.

EXAMPLE 1



Height in function of time of flight

EXAMPLE 1



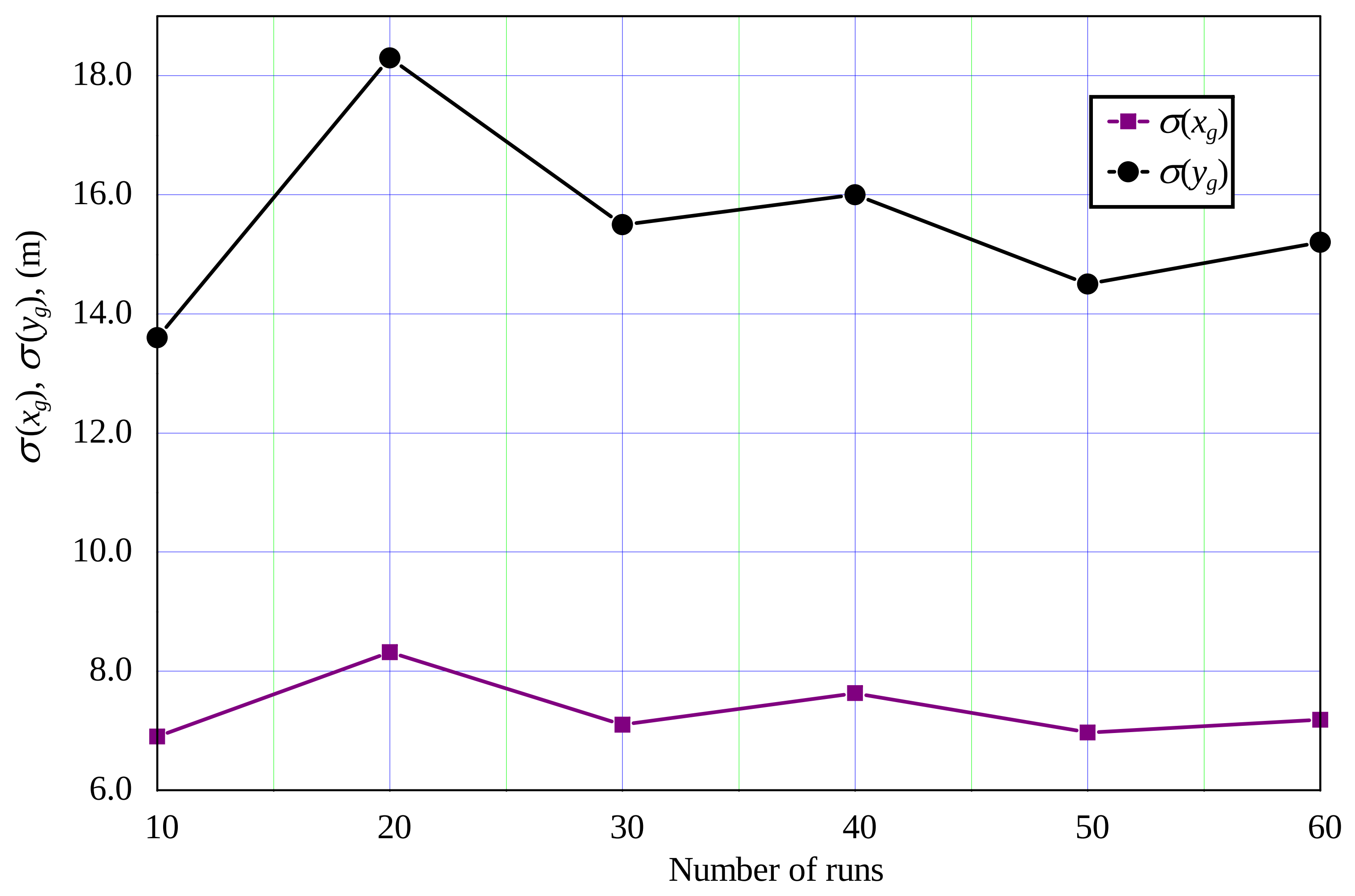
Influence of Earth rotation to deviation in y_g direction

EXAMPLE 2

Influence of the instrumental errors – Monte Carlo simulation

In order to obtain proper statistics on target by Monte-Carlo method number of simulations (number of runs) should large enough. It could be obtain by varying number of runs as it is shown on figure, where, calculated standard deviation of the coordinates of impact point due to gyro in-run error =0.0001 s^{-1} is presented. It can be seen that after 30 runs standard deviations do not change much. So, for the further simulation we will use that number of runs.

EXAMPLE 2



Influence of number of runs in Monte Carlo simulation

EXAMPLE 2

Influence of various errors

Simultaneous contributions of all errors are presented on the next two diagrams. The statistical characteristics of the errors are:

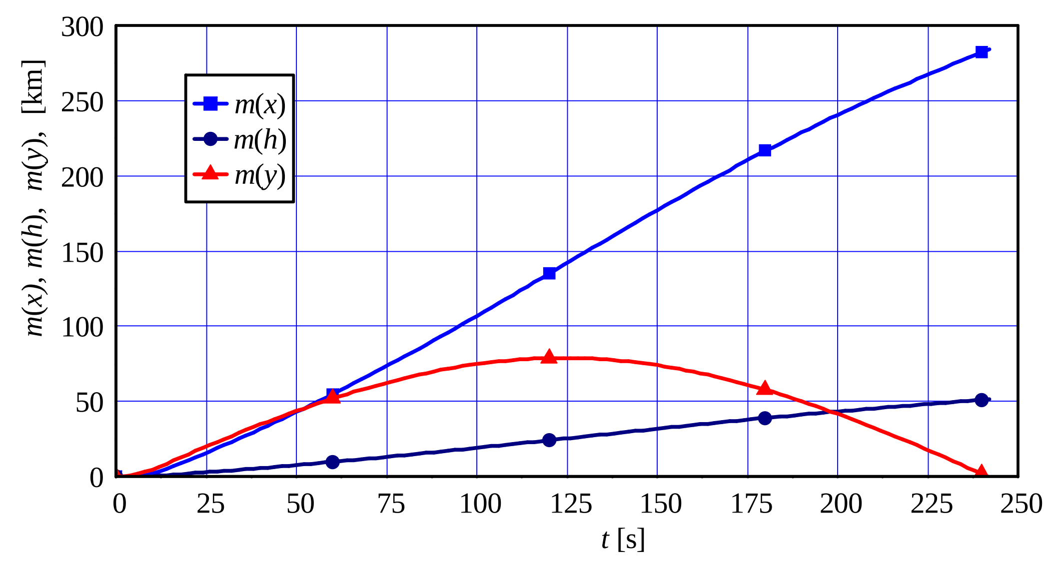
- rate gyros scale factor standard deviation = 0.05 %
- rate gyros bias standard deviation = 5 º/h
- rate gyros noise standard deviation = 180 º/h
- accelerometer scale factor standard deviation = 0.05 %
- accelerometer bias standard deviation = 0.5 mg
- accelerometer noise standard deviation = 50 mg

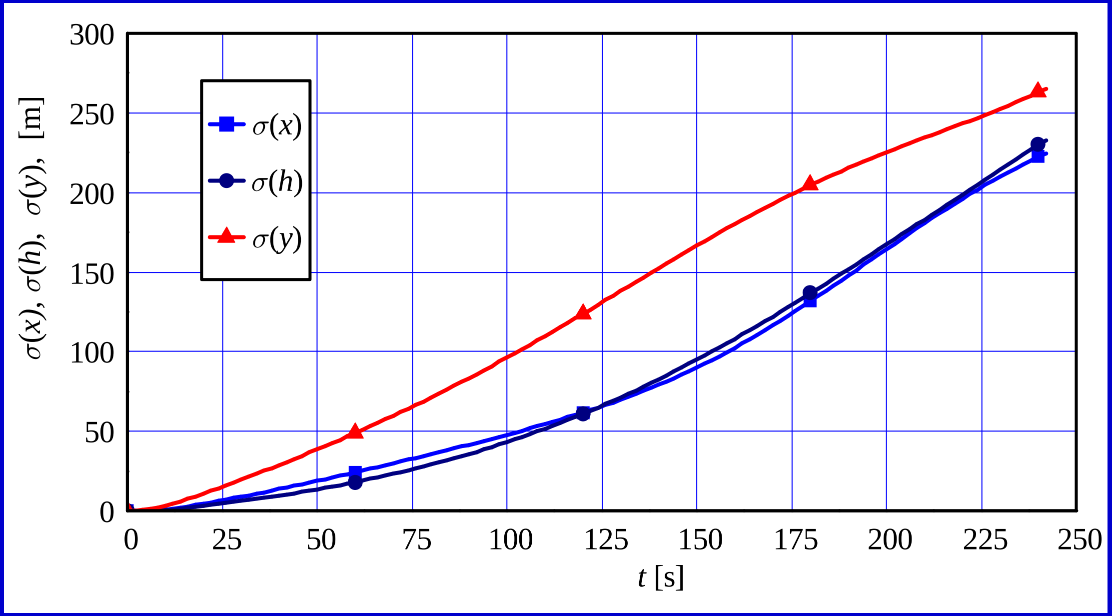
Results of calculation are shown on the next diagrams

EXAMPLE 2

Influence of various errors

Mean values of co-ordinates vs. time





Root mean square values of coordinates vs. time

EXAMPLE 3

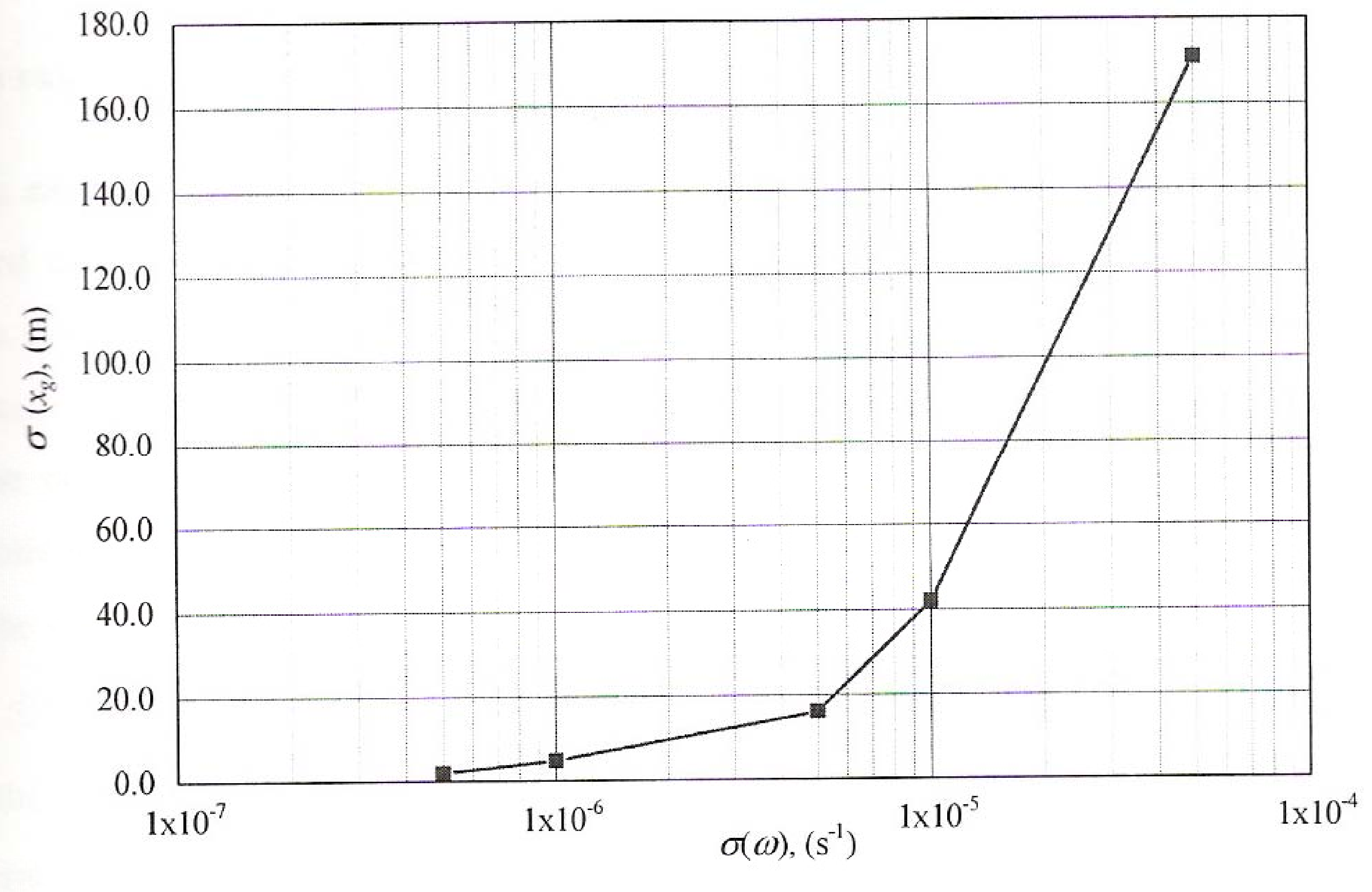
Influence of gyro errors on the accuracy of co-ordinates

Bias errors

- To examine influence of bias error (fixed bias, g-independent), bias was added to all of the three gyros measurement (values obtained from reference trajectory). During one trajectory bias was constant. Thirty trajectories were calculated with generated biases with one (same) value of bias standard deviation. Then new value of standard deviation was chosen and new set of 30 trajectories were calculated. After that the coordinates of the impact point were analyze in statistical sense. Following this procedure standard deviation of x_g and y_g are obtained and showed on diagrams.
- It can be seen significant rise of standard deviation start at 1×10^{-6} s^{-1}. If we allow standard deviations of x_g and y_g up to 50 m, we can read out from diagrams that gyro bias is equal or less than 5×10^{-6} s^{-1}, or one degree per hour.

EXAMPLE 3

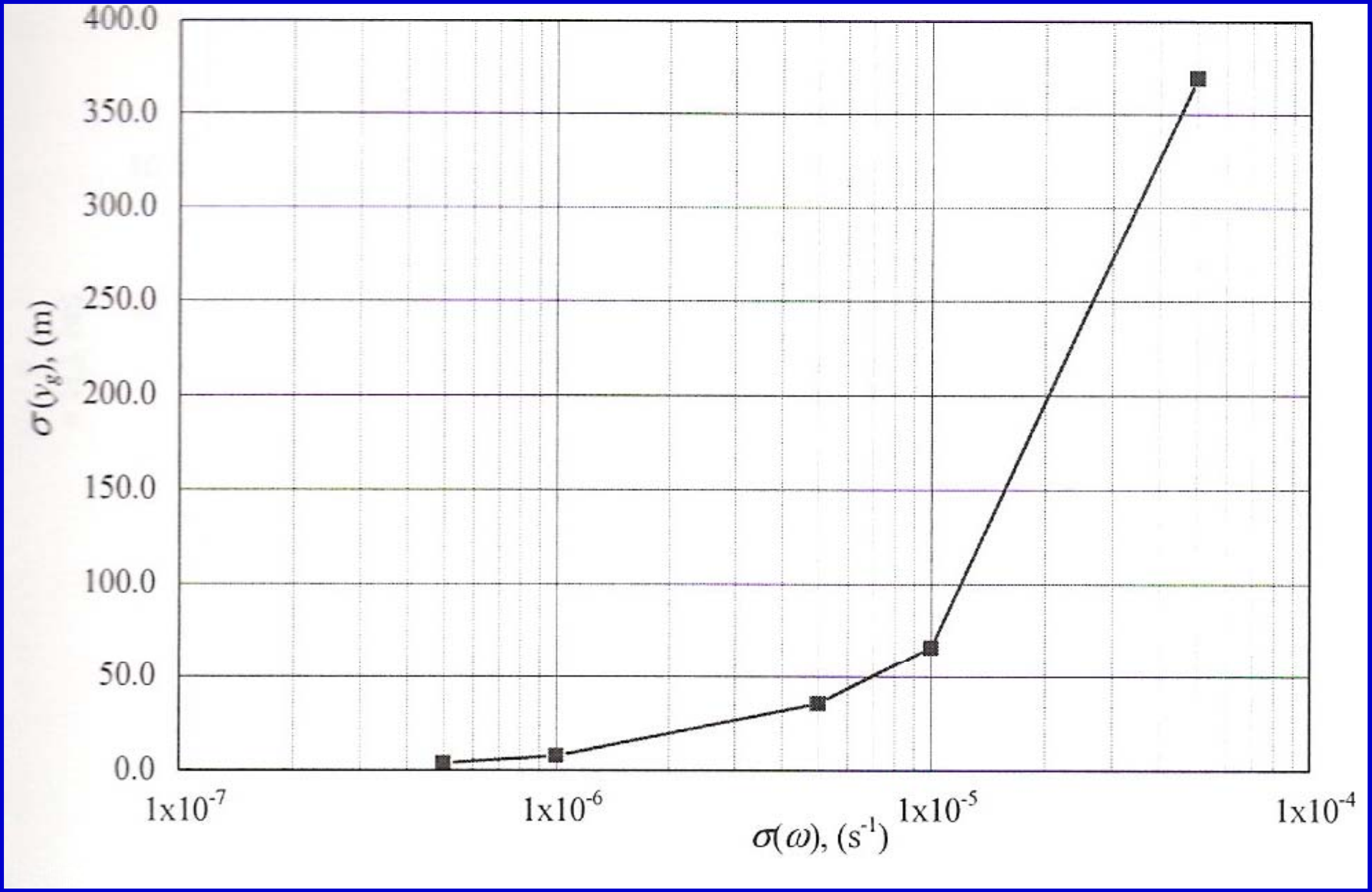
Influence of gyro errors on the accuracy of co-ordinates



Influence of gyro bias to deviation in x_g direction

EXAMPLE 3

Influence of gyro errors on the accuracy of co-ordinates



Influence of gyro bias to deviation in y_g direction

EXAMPLE 3

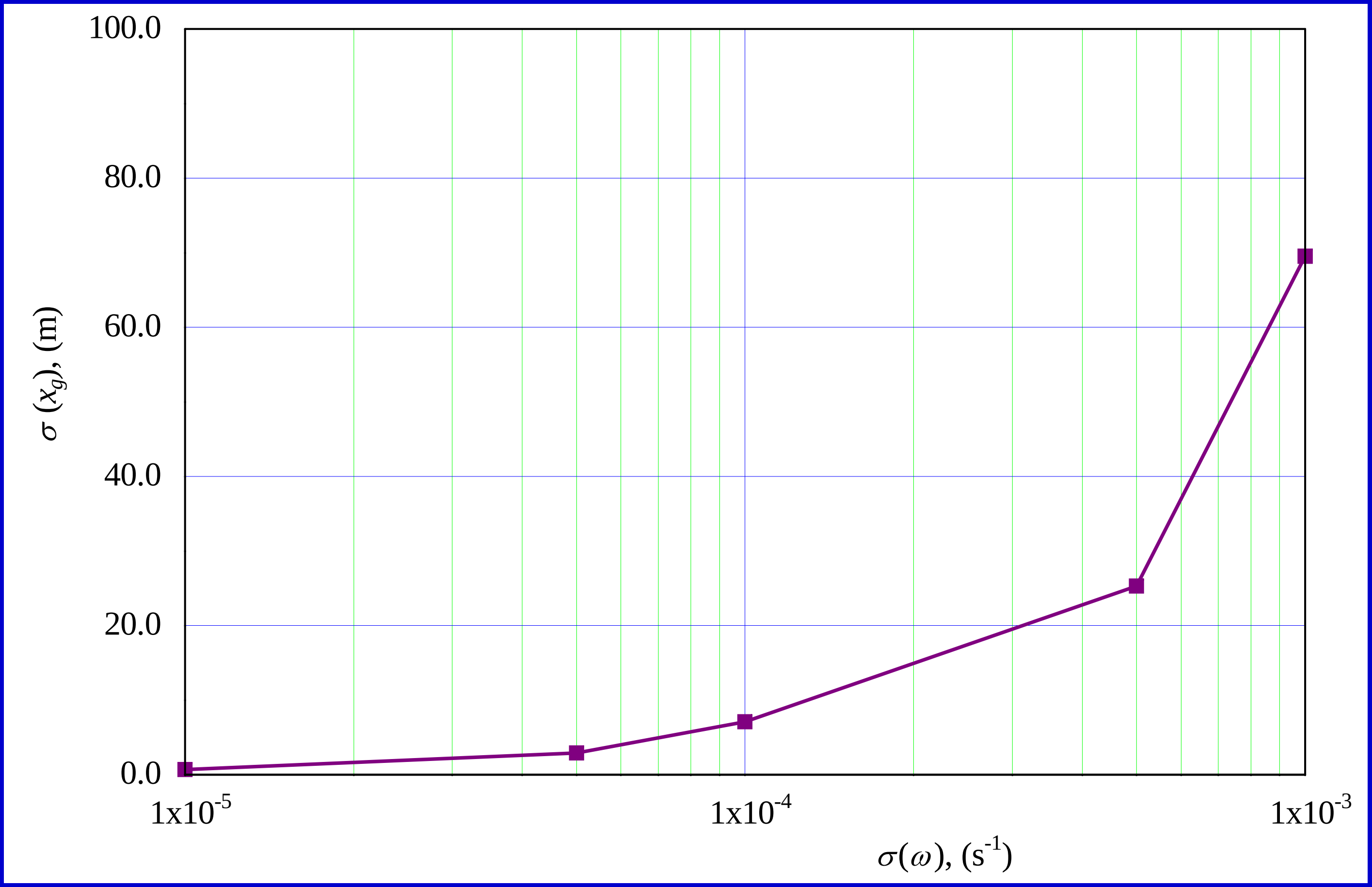
Influence of gyro errors on the accuracy of co-ordinates

In-run random errors

- To examine influence of in-run random error generated random numbers with specified standard deviation were added to all of the three gyros measurement along the trajectory (values obtained from reference trajectory). Thirty trajectories were calculated with different generated sequence of pseudo random numbers with same value of standard deviation. After that the coordinates of the impact point were analyze in statistical sense. Following this procedure standard deviation of x_g and y_g are obtained and showed on diagrams.
- It can be seen significant rise of standard deviation start at $1\times10^{-4}\ s^{-1}$. If we allow standard deviations of x_g and y_g up to 50 m, we can read out from diagrams that gyro in-run error is equal or less than 5×10^{-4} s^{-1}, or one hundred degree per hour.

EXAMPLE 3

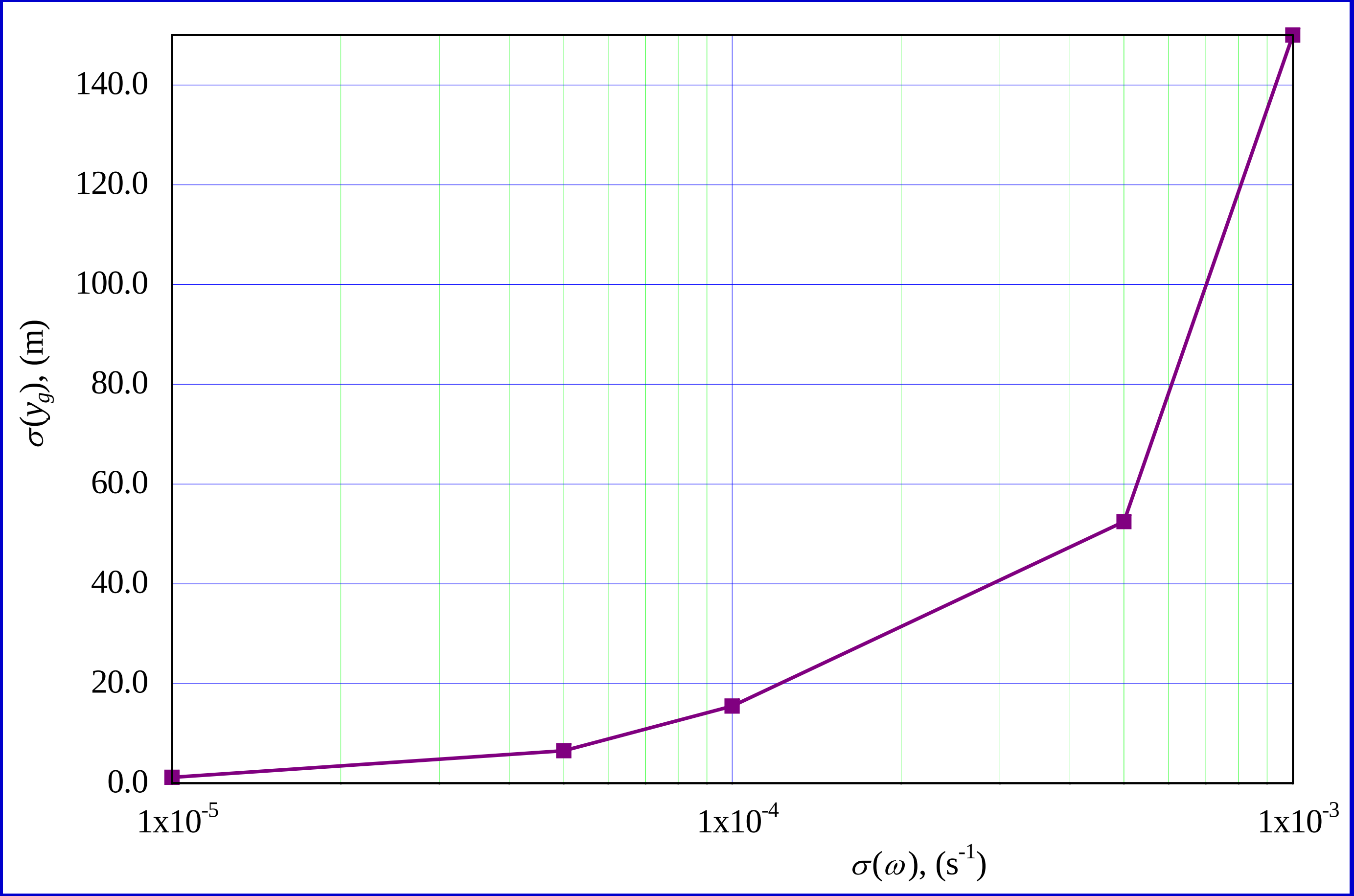
Influence of gyro errors on the accuracy of co-ordinates



Influence of gyro in-run errors to deviation in x_g direction

EXAMPLE 3

Influence of gyro errors on the accuracy of co-ordinates



Influence of gyro in-run errors to deviation in y_g direction

EXAMPLE 4

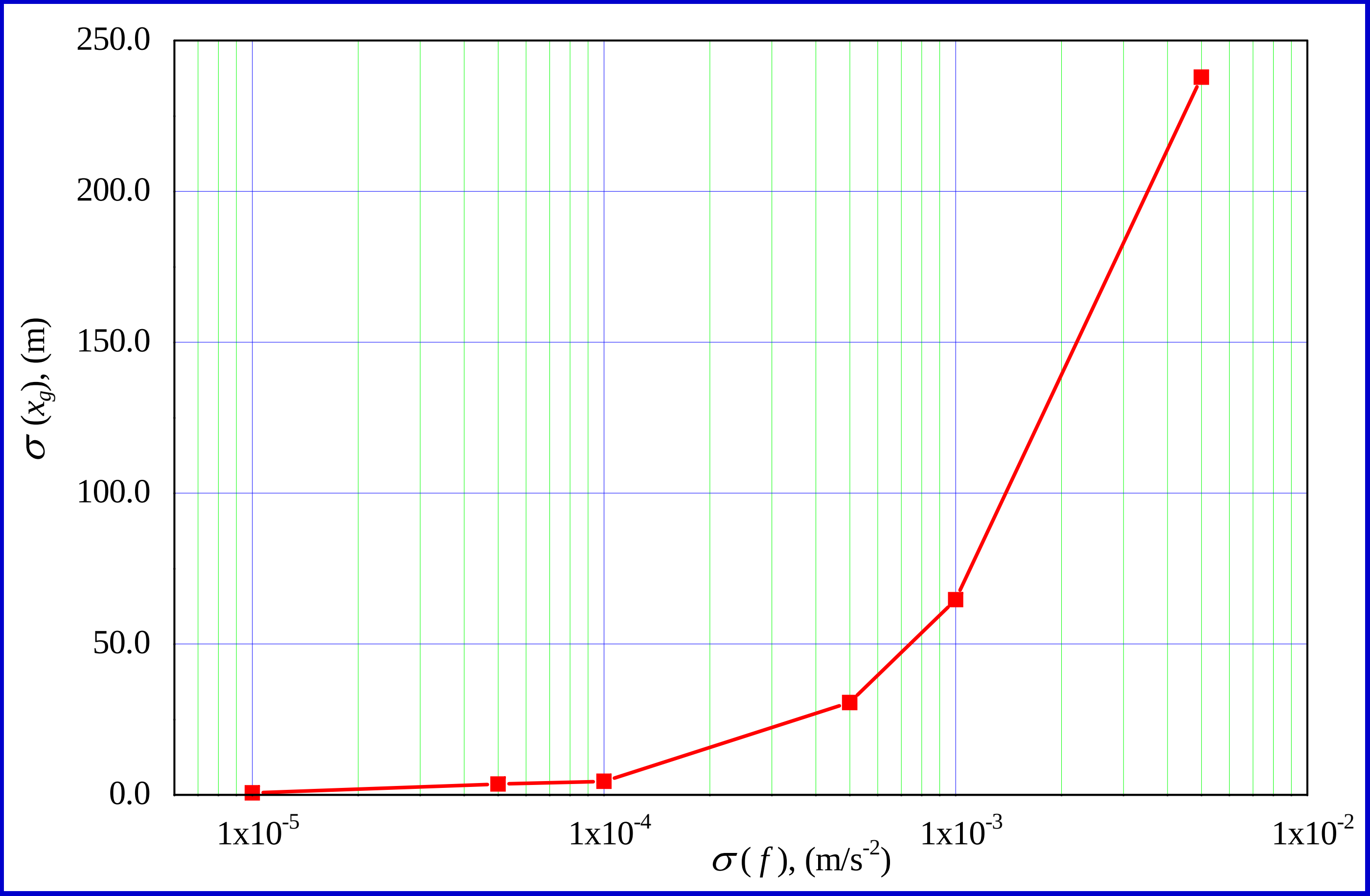
Influence of accelerometers errors on the accuracy of co-ordinates

Bias errors

- To examine influence of accelerometer bias error (fixed bias), bias was added to all of the three accelerometer measurement (values obtained from reference trajectory). During one trajectory bias was constant. Thirty trajectories were calculated with generated biasis with one (same) value of bias standard deviation. Then new value of standard deviation was chosen and new set of trajectory (30) were calculated. After that the coordinates of the impact point were analyze in statistical sense. Following this procedure standard deviation of xg and yg are obtained and showed on diagrams.
- It can be seen significant rise of standard deviation start at 1×10^{-3} m/s^{-2}. If we allow standard deviations of x_g and y_g up to 50 m, we can read out from diagrams that accelerometer bias is equal or less than 1×10^{-3} m/s^{-2}, or 0.01 g.

EXAMPLE 4

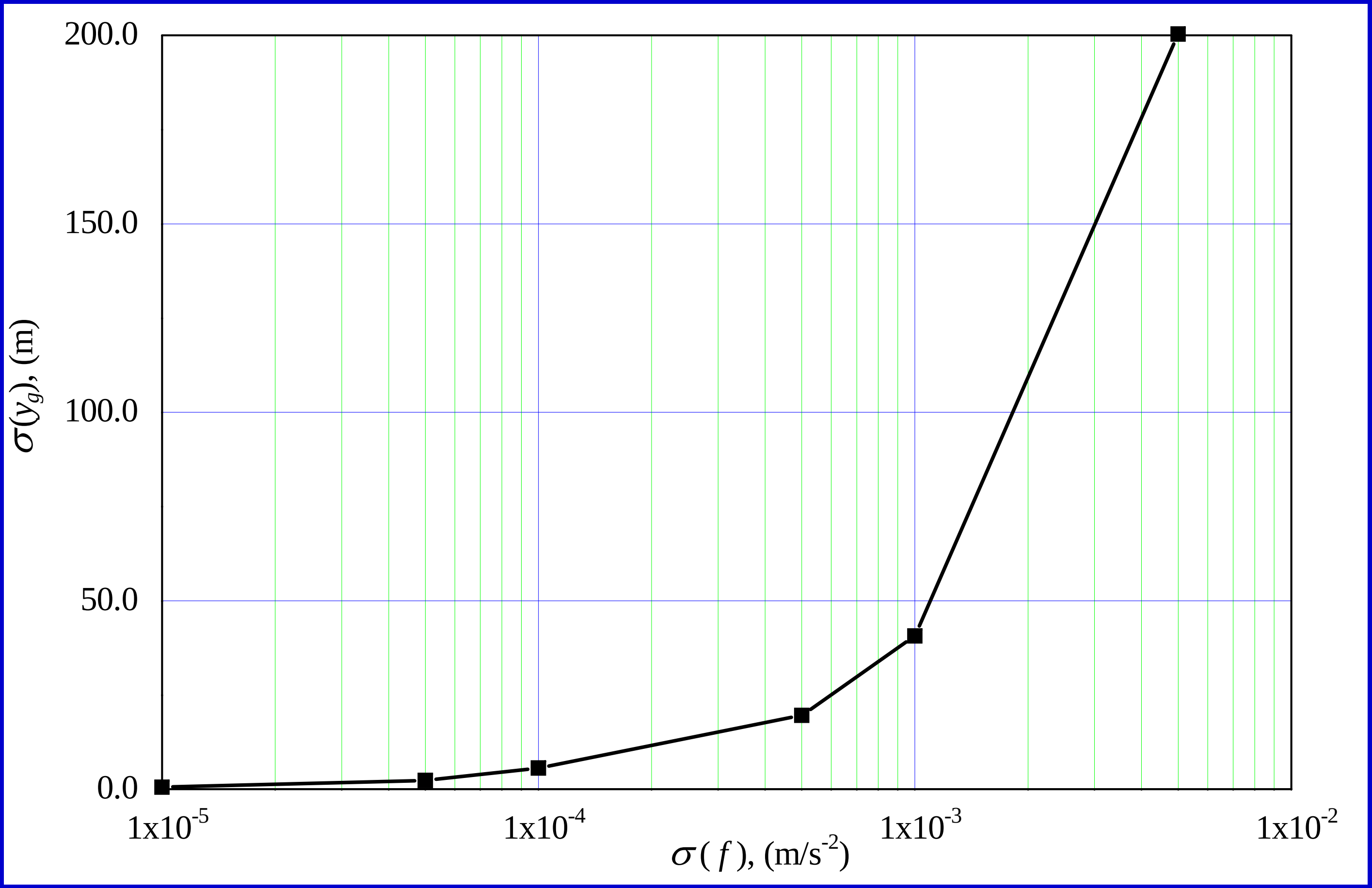
Influence of accelerometers errors on the accuracy of co-ordinates



Influence of accelerometer bias to deviation in x_g direction

EXAMPLE 4

Influence of accelerometers errors on the accuracy of co-ordinates



Influence of accelerometer bias to deviation in y_g direction

EXAMPLE 4

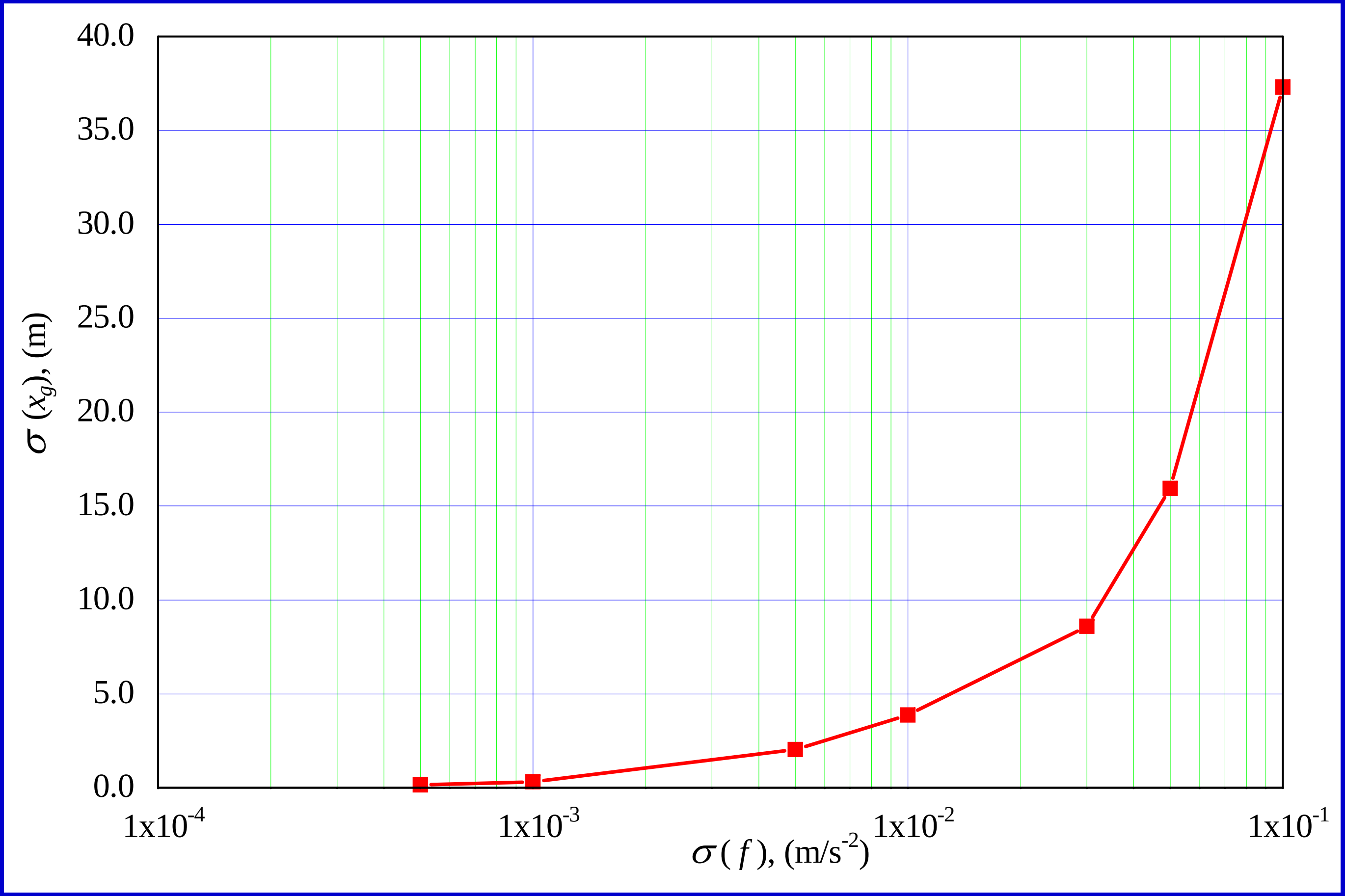
Influence of accelerometers errors on the accuracy of co-ordinates

In-run random errors

- To examine influence of accelerometer in-run random error, generated random numbers with specified standard deviation were added to all of the three accelerometer measurement along the trajectory (values obtained from reference trajectory). Thirty trajectories were calculated with different generated sequence of pseudo random numbers with same value of standard deviation. After that the coordinates of the impact point were analyze in statistical sense. Following this procedure standard deviation of x_g and y_g are obtained and showed on diagrams.
- It can be seen significant rise of standard deviation start at 2×10^{-2} m/s^{-2}. Compare the influence of accelerometer in-run error with bias error we can see that influence of in-run error is much less than the influence of bias error.

EXAMPLE 4

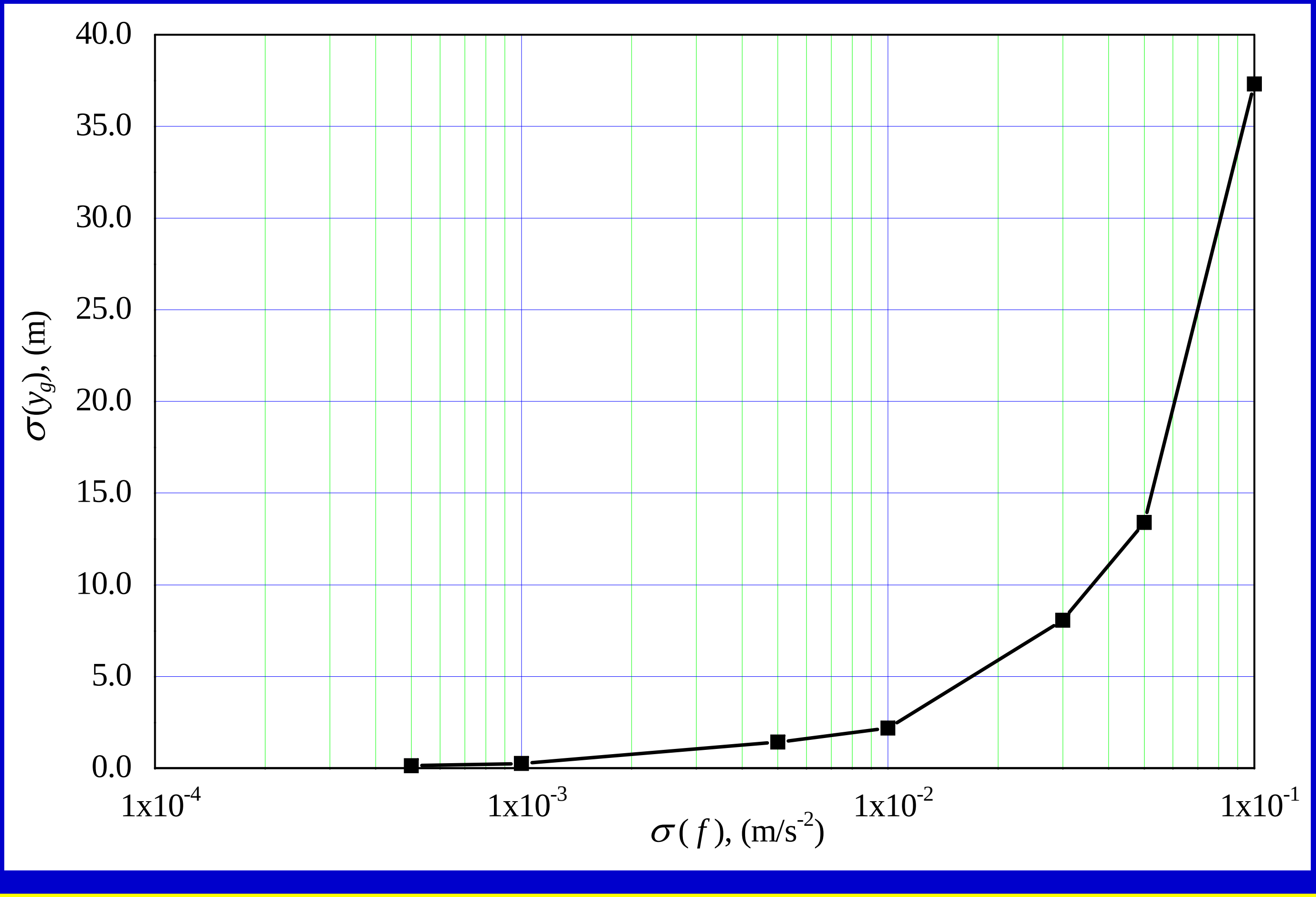
Influence of accelerometers errors on the accuracy of co-ordinates



Influence of accelerometer in-run random errors to deviation in x_g direction

EXAMPLE 4

Influence of accelerometers errors on the accuracy of co-ordinates



Influence of accelerometer in-run random errors to deviation in y_g direction